

Final Project

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Abstract and Summary

0.1 Abstract

This project mainly analyzes 5 years of monthly closing data from October 2014 through October 2019.

0.2 Data descriptions

a. S&P 500 index (VFINX)

The Standard & Poor's 500 Index, or S&P 500 Index, is a stock Index that records 500 publicly traded companies in the United States. This stock index is created and maintained by Standard & Poor's. All the companies covered by the S&P 500 are listed on major U.S. exchanges, such as the New York Stock Exchange and Nasdaq. The S&P 500 contains more companies than the Dow, so risk is more diversified and can reflect broader market changes.

b. European stock index (VEURX)

Vanguard European Stock Index Fund seeks to track the performance of a benchmark index that measures the investment return of stocks issued by companies that are mostly located in European.

c. Emerging markets fund (VEIEX)

Vanguard Emerging Markets Stock Index Fund seeks to track the performance of a benchmark index that measures the investment return of stocks issued by companies located in emerging market countries.

d. Long term bond index (VBLTX)

This index fund offers a low-cost, diversified way to invest in bonds with a wide range of U.S. investment-grade bonds with maturities longer than 10 years. About 60 percent of the fund's assets are invested in corporate bonds and 40 percent in US government bonds over that maturity horizon.

e. Short term bond index (VBISX)

Vanguard Short-Term Bond Index Fund seeks to track the performance of a market-weighted bond index with a short-term dollar-weighted average maturity.

f. Pacific stock index (VPACX)

Vanguard Pacific Stock Index Fund seeks to track the performance of a benchmark index that measures the investment return of stocks issued by companies in the major markets of the Pacific region.

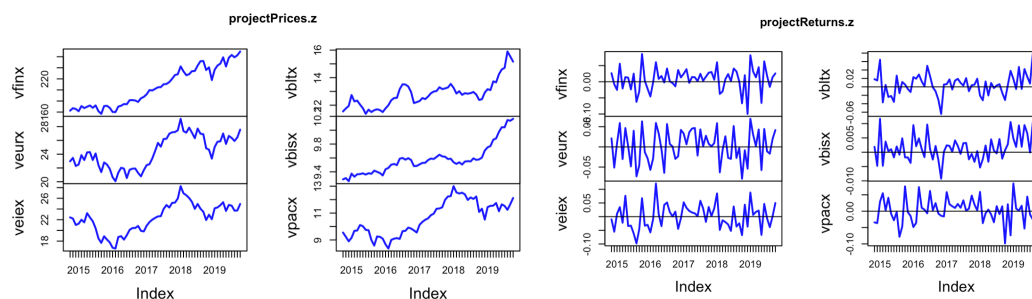
0.3 Main Findings

- In general in the projectPrices plot, all six mutual funds have similar trends. Especially for the European stock index (VEURX), Emerging markets fund (VEIEX), and Pacific stock index (VPACX), all of them fluctuated first and met a small peak around 2015, then started to decrease until they reached the trough in 2016. After 2016, their general trends are increasing until the peak in 2018.
- On the Return side, the European stock index (VEURX), Emerging markets fund (VEIEX), and Pacific stock index (VPACX) also have similar trends that have generally bigger fluctuations than others.

- VFINX has the most outstanding normal return and VBISX has the least normal return mean. VEIEX has the most standard deviation and VBISX has the least.
- The empirical value-at-risk is relatively closed to the estimated VaR, thus the estimation above is relatively precise.

Return Calculation

1.1 Time Plots of Monthly Prices and Returns

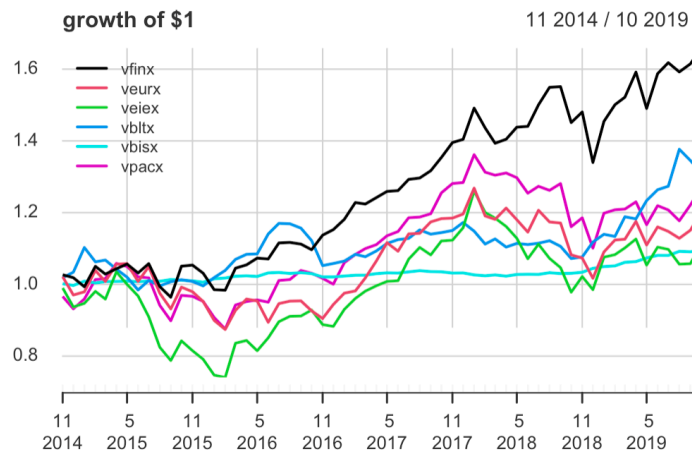


In general in the projectPrices plot above, all six mutual funds have similar trends.

Especially for the European stock index (VEURX), Emerging markets fund (VEIEX), and Pacific stock index (VPACX), all of them fluctuated first and met a small peak around 2015, then started to decrease until they reached the trough in 2016. After 2016, their general trends are increasing until the peak in 2018. After the peak, they started to decrease until the end of 2018, and what's following was fluctuation to the end.

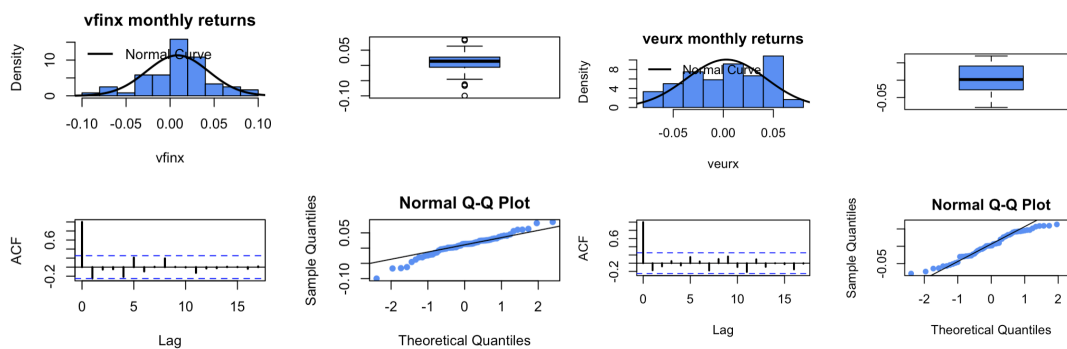
On the Return side, the European stock index (VEURX), Emerging markets fund (VEIEX), and Pacific stock index (VPACX) also have similar trends that have generally bigger fluctuations than others. Meanwhile, VBISX" and "VBLTX are in similar movement trends as well with less fluctuation which their prices were steadily upwards. One event that may account for the trough of 2018 in VPACX is the trade war starting at 2018, which in

some way restricted the free market environment in international trade which may affects the funds market throughout the pacific countries.



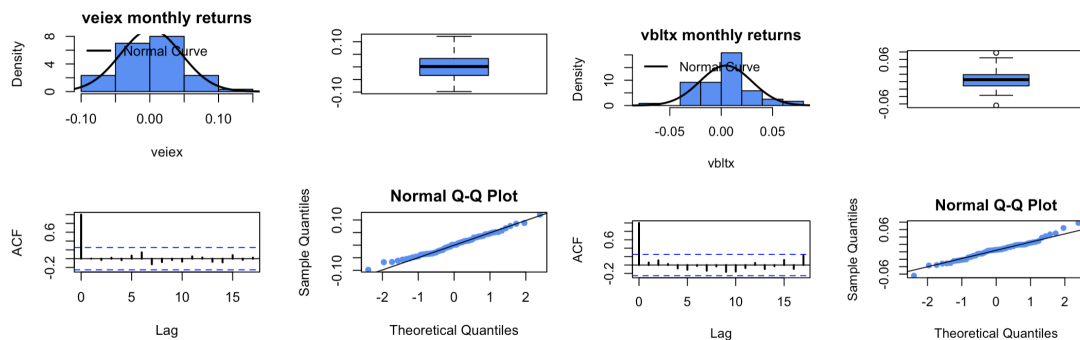
According to the graph above, obviously, the VFINX has the outstanding highest future value among the six mutual funds. The VBISX had a steady trend over these five years. The other four had similar trends and the VEIEX has the lowest future value among these four mutual funds. I am not that surprised since the VFINX stands for the S&P 500 index, whose stocks are closer to “No-Question-Asked” properties, and safe. That might attract more investments and thus generates more future return.

1.2 Normality of Continuously Compounded Returns



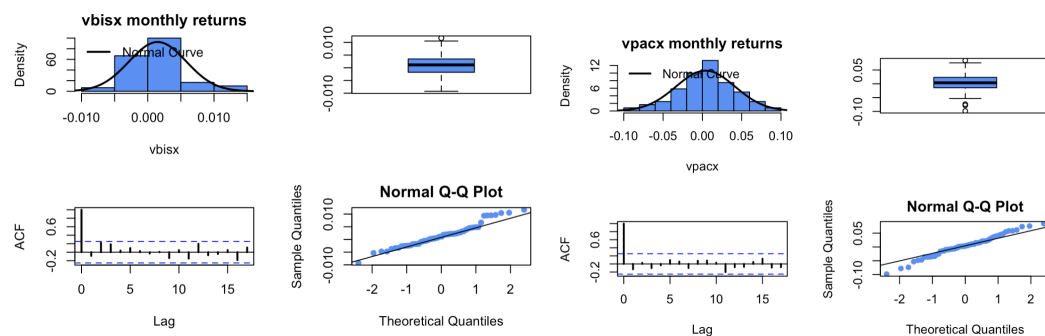
The histogram of VFINX Monthly returns is normally distributed and symmetric, with a peak in the middle around 0.00. The box plot seems normal, with several outliers. Most dots are on the straight line in Q-Q plot with several outliers away from 0.

The histogram of VEURX Monthly returns is not normally distributed, with almost even splits in all data chunks without a peak in the middle. The box plot seems that there are a lot of outliers. Most dots are not really located on the straight line in Q-Q plot with certain outliers away from 0.



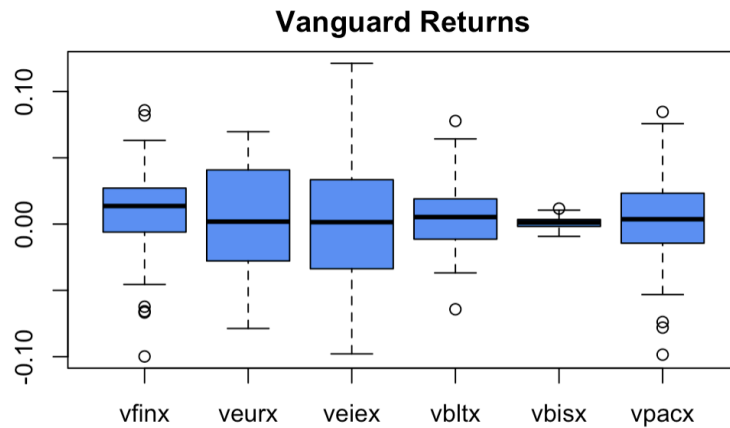
The histogram of VEIEX Monthly returns is normally distributed and a little bit right skewed, with a peak in the middle less than 0.00. The box plot seems normal, with almost no outliers. Most dots are on the straight line in Q-Q plot with several outliers away from 0.

The histogram of VBITX Monthly returns is normally distributed and symmetric, with a peak in the middle of more than 0.00. The box plot seems normal, with several outliers. Most dots are on the straight line in Q-Q plot with several outliers away from 0.



The histogram of VBISX Monthly returns is normally distributed and a little bit right skewed, with a peak in the middle of more than 0.00. The box plot seems normal, with limited outliers. Most dots are slightly above the straight line in Q-Q plot with several outliers away from 0, especially the part bigger than 1.

The histogram of VPACX Monthly returns is normally distributed and symmetric, with a peak in the middle around 0.00. The box plot seems normal, with several outliers, especially at the lower range. Most dots are on the straight line in Q-Q plot with several outliers away from 0.



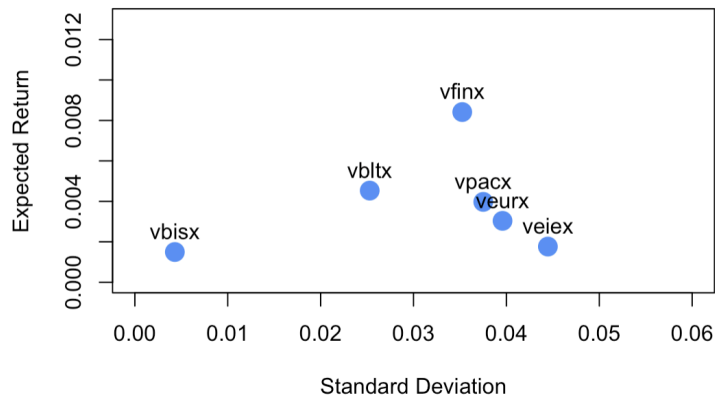
Among the above boxplot, we could conclude that VBLTX, VBISX, and VPACX have the mean near 0, and VFINX above 0 while VEURX and VEIEX are below 0. Also, the range of VEIEX is the biggest, the VBISX is the smallest. The VFINX has the most outliers while VEURX and VEIEX have the least.

1.3 Descriptive Statistics

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.00841434	0.00303641	0.00176488	0.00453187	0.00149611	0.00397383

Std Dev	0.03524581	0.03958606	0.04446615	0.02528675	0.00430548	0.03750994
Skewness	-0.5206138	-0.226552	0.26472198	0.20836701	0.42022992	-0.2351863
Excess Kurtosis	0.87697662	-0.969996	-0.2469819	0.80210757	0.19330558	0.23589311
1% Quantile	-0.0802823	-0.0753216	-0.0800368	-0.0480491	-0.0069302	-0.0864588
5% Quantile	-0.06249	-0.0631494	-0.0619962	-0.0319954	-0.0047404	-0.054188

VFINX has the most outstanding normal return and VBISX has the least normal return mean. VEIEX has the most standard deviation and VBISX has the least, which is consistent with the conclusion we obtained from the last part. The VBLTS has the least skewness, so it is the most normally distributed, and VFINX is the least normally distributed since it has the highest skewness. Besides, VFINX and VEURX have the highest excess kurtosis which means they may have a larger range of combined size than others.



VFINX has the highest average return and VBISX has the least normal return mean, which is also consistent with the data table above. VEIEX has the most standard deviation and VBISX has the least, which is consistent with the conclusion we obtained from the last

part. VBISX seems to be most normally distributed. This makes sense since VBISX is a government short-term bond index which its return should not vary too much.

1.4 Sharpe's Ratio

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Sharpe's Ratio	0.2269113	0.06617832	0.03031994	0.1627416	0.2507149	0.09483252
SE	0.139069	0.1315039	0.132023	0.1402013	0.125763	0.133604

The VBISX has the largest Sharpe Ratio with the highest slope, and its standard error is about half of its original value, so it is quite a precise estimation. In general, VEURX, VEIEX, VBLTX, and VPACX are not estimated well since its standard error is almost larger than the original value.

1.5 Estimated standard errors and formed 95% confidence intervals

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.00841434	0.00303641	0.00176488	0.00453187	0.00149611	0.00397383
SE mean	0.00455022	0.00511054	0.00574056	0.0032645	0.00055583	0.00484251
95% higher	0.01751477	0.01325748	0.01324599	0.01106088	0.00260778	0.01365886
95% lower	-0.0006861	-0.0071847	-0.0097162	-0.0019971	0.00038444	-0.0057112

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
SD	0.03524581	0.03958606	0.04446615	0.02528675	0.00430548	0.03750994
SE SD	0.00321749	0.0036137	0.00405919	0.00230835	0.00039303	0.00342417

95% higher	0.04168079	0.04681345	0.05258452	0.02990345	0.00509154	0.04435829
95% lower	0.02881084	0.03235867	0.03634778	0.02067004	0.00351941	0.0306616

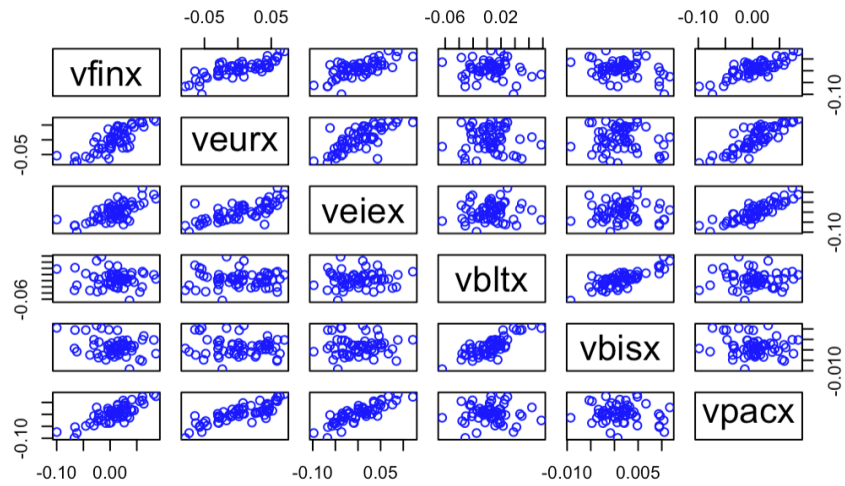
All these means and standard deviations are estimated precisely. The standard deviation is estimated more precisely than the mean because almost all standard errors of standard deviation have a smaller data interval than the mean. Moreover, standard errors of standard deviation have all positive numbers which imply a more accurate estimation.

1.6 Annual Sharpe's Ratio

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.00841434	0.00303641	0.00176488	0.00453187	0.00149611	0.00397383
Mean. Annual	0.10097207	0.03643687	0.02117854	0.05438249	0.01795337	0.04768596
SD	0.03524581	0.03958606	0.04446615	0.02528675	0.00430548	0.03750994
SD. Annual	0.12209508	0.13713012	0.15403526	0.08759586	0.0149146	0.12993826
Sharpe's Ratio. Annual	0.78604369	0.22924848	0.10503138	0.56375369	0.86850237	0.32850956
\$1 Growth after 5 years	1.656754122	1.199835375	1.11170256	1.312472076	1.093919196	1.2692546

According to the table, the annual Sharpe ratio is still the same ranking as the monthly Sharpe ratio. After five years, VFINX has the most expected return while VBISX, which is consistent with the conclusion we derived from the mean.

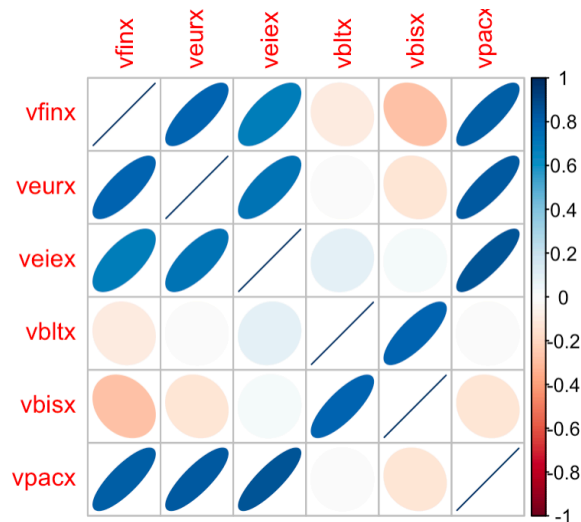
1.7 Covariance matrix of the returns



	vfinx	veurx	veiex	vbltx	vbisx	vpacx
vfinx	0.001242267	1.099657	0.00107615	-8.36356E-05	-4.1119E-05	0.0010781
veurx	0.001099657	0.001567054	0.00129763	-6.73076E-07	-2.251E-05	0.00123258
veiex	0.001076146	0.001297628	0.00197723	0.000107794	7.57001E-06	0.0014181
vbltx	-8.36356E-05	-6.7308E-07	0.00010779	0.000639421	8.69034E-05	-5.716E-06
vbisx	-4.11186E-05	-2.251E-05	7.57E-06	8.69034E-05	1.85366E-05	-1.993E-05
vpacx	0.001078098	0.001232575	0.0014181	-5.716E-06	-1.9929E-05	0.00140699

According to the results above, the covariance between VBLTX and VFINX, VPACX, VEIEX, VEURX and between VIBSX and VFINX is negative indicating a strong negative linear association. The covariance between VBLTX and VIBSX, and between VEIEX, and VPACX are positive, indicating positive linear associations.

1.8 correlation matrix of the returns



	vfinx	veurx	veiex	vbltx	vbisx	vpacx
vfinx	1	0.788149094	0.68664992	-0.093840681	-0.27097114	0.81546601
veurx	0.788149094	1	0.73718997	-0.00067323	-0.1320685	0.83009187
veiex	0.686649917	0.737189969	1	0.09586604	0.03954955	0.85021947
vbltx	-0.093840681	-0.00067323	0.09586604	1	0.798229114	-0.0060265
vbisx	-0.27097114	-0.1320685	0.03954955	0.798229114	1	-0.1233922
vpacx	0.815466008	0.830091868	0.85021947	-0.006026501	-0.12339217	1

Most correlations are positive according to the result, except for the relationship between VBLTX and VFINX, VBLTX and VEURX, VBISX and VEURX, VPACX and VBISX, and VPACX and VBLTX, which are negative relationships. The general relationships are moderately strongly related but the relationship between VBLTX with other assets and VBISX with other assets is very weak. Among those relationships, VPACX and VEIEX are the most correlated, while VBLTX and VEURX are the least.

Diversification could certainly reduce the risk of investments that were somehow shown through the data. However, it benefits more from the equity securities who already realized within the portfolio.

Value-at-Risk Calculations

2.1 Normal Value-at-Risk over Different Investment Horizons

VaR of \$100,000 Over a one month investment Horizo

	VaR 1%	Var 5%	SE	95% CI lower	95% CI upper
vfinx	-7093.789	-4835.182	866.5101	-6598	-3202
veurx	-8520.4316	-6018.9354	702.8285	-7477	-4722
veiex	-9668.0396	-6888.7739	717.352	-8376	-5564
vbltx	-5284.6295	-3638.2764	507.9982	-4696	-2705
vbisx	-848.3728	-557.0192	74.612	-711.3	-4188
vpacx	-7991.3278	-5609.0078	792.3061	-7254	-4148

At the 95% confidence level, all the results are negative, which may imply that the range is relatively small, and the estimation is quite precise. VEIEX has the highest VaR and VPACX has the lowest.

2.2 VaR of \$100,000 over a year investment horizon

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.008414339	0.003036406	0.00176488	0.004531874	0.001496114	0.00397383
Mean. Annual	0.100972068	0.036436872	0.02117854	0.054382488	0.017953368	0.04768596
SD	0.035245814	0.039586057	0.04446615	0.025286746	0.004305475	0.03750994
SD. Annual	0.122095081	0.137130124	0.15403526	0.087595858	0.014914603	0.12993826
1% VaR	-16728.478	-24616.028	-28620.613	-13877.196	-1660.381	-22477.01
5% VaR	-9503.2704	-17231.4574	-20720.208	-8579.449	-655.7383	-15299.058

According to the result, VBISX has the highest VaR and the VEIEX has the lowest.

2.3 Normal Value-at-Risk over Different Investment Horizons

	VaR 1%	Var 5%
vfinx	-7714.4174	-6057.7522

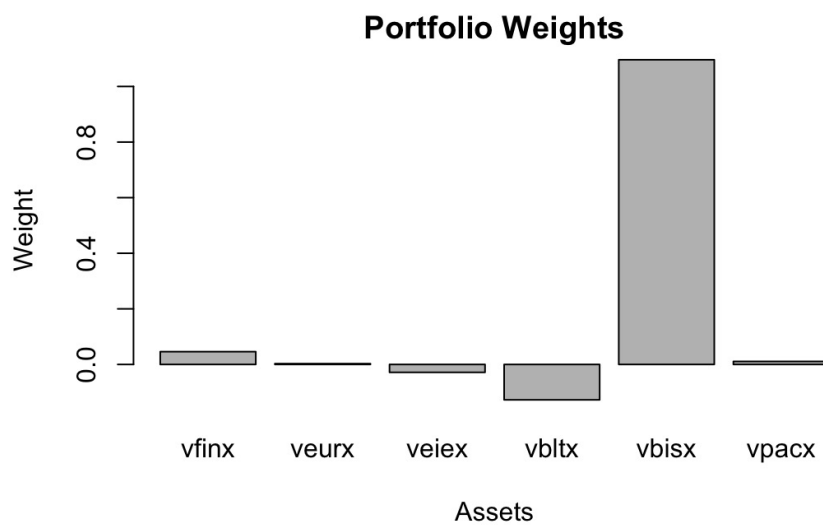
veurx	-7255.482	-6119.677
veiex	-7691.7588	-6011.3517
vbltx	-4691.2995	-3148.8944
vbisx	-690.6204	-472.9168
vpacx	-8282.6702	-5274.5968

The empirical value-at-risk is relatively closed to the estimated VaR, thus the estimation above is relatively precise.

Portfolio Theory

3.1 Global Minimum Variance Portfolio with Short Sales

Portfolio Sharpe Ratio		0.3946273				
Portfolio SD		0.002620859				
Portfolio expected return		0.001450929				
Portfolio weights	vfinx	veurx	veiex	vbltx	vbisx	vpacx
	0.0459	0.0028	-0.0284	-0.1273	1.0961	0.0109



As the result shows, both VEURX and VEIEX have negative weights in the global minimum variance portfolio.

3.2 Annual Global Minimum Variance Portfolio with Short Sales

Annual Portfolio SD	0.009078922
Annual Portfolio expected return	0.017411148
Annual Portfolio Sharpe Ratio	1.367028833

According to the table, the annual Sharpe ratio is still the same proportion size as the monthly Sharpe ratio.

3.3 Value-at-Risk of investment over a one-month investment horizon

	VaR 1%	Var 5%
vfinx	-7358	-4956
veurx	-8905	-6208
veiex	-10168	-7138
vbltx	-5429	-3706
vbisx	-852	-559
vpacx	-8329	-5772

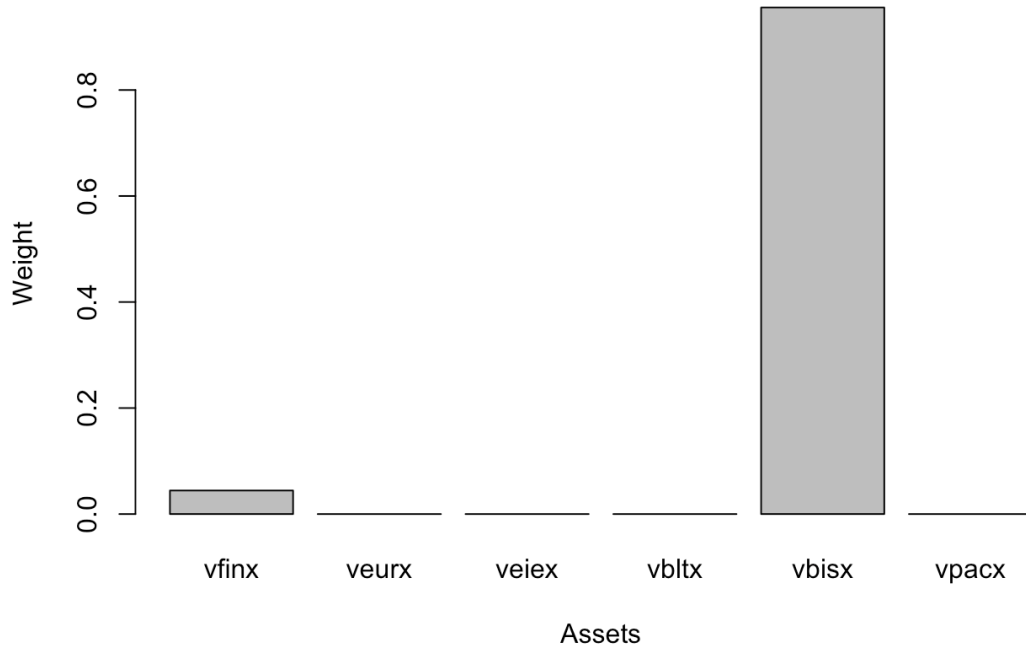
The general results are similar to the VaR values for the individual assets.

3.4 Global Minimum Variance Portfolio with NO Short Sales

Portfolio SD	0.00399
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Portfolio expected return		0.0018				
Portfolio weights	vfinx	veurx	veiex	vbltx	vbisx	vpacx
	0.0444	0	0	0	0.9556	0

Portfolio Weights



Annual Portfolio SD	0.013821765
Annual Portfolio expected return	0.0216
Annual Portfolio Sharpe Ratio	1.201004319

A worldwide portfolio has a higher Sharpe ratio proportion than without short sales, which implies the short sale might be beneficial.

3.5 Value-at-Risk of investment without short sales over a one-month investment

horizon

VaR 5%	-286
VaR 1%	-465
VaR 5% NS	-475
VaR 1% NS	-747

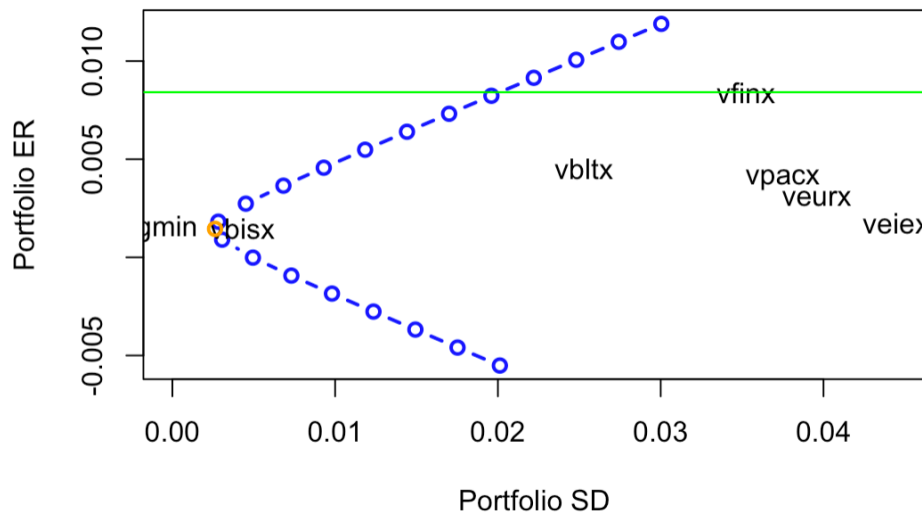
Var of global minimum portfolio without shorts is higher than Var of global minimum portfolio with shorts in absolute value. I am not surprised as it involves more risks.

3.6 Value-at-Risk of investment without short sales over a one-month investment horizon

horizon

Portfolio expected return		0.0201				
Portfolio SD		0.00841				
	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Portfolio weights	0.907	-0.329	-0.282	0.41	0.302	-0.009

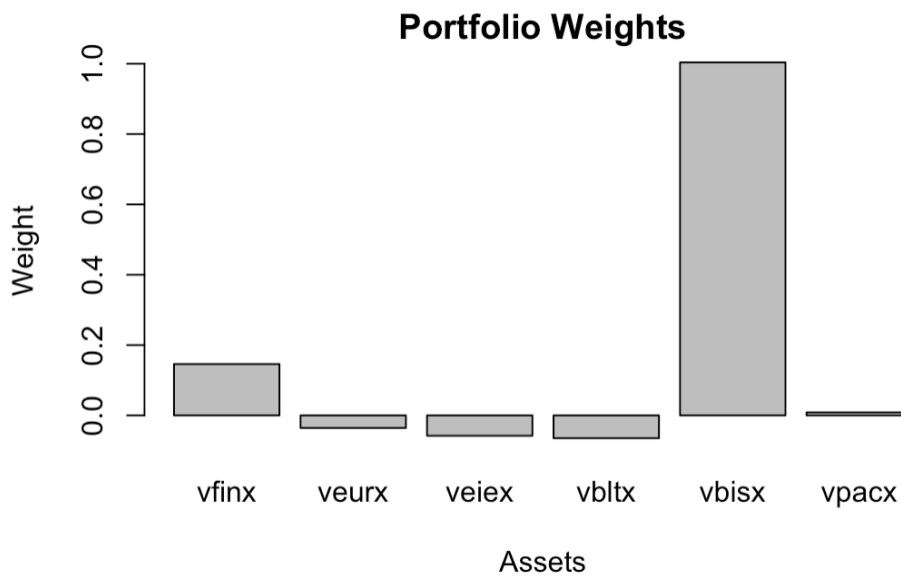
Efficient Frontier



Except for VBISX, the other five assets are away from the efficient frontier, and VBISX is relatively closed to the global minimum point.

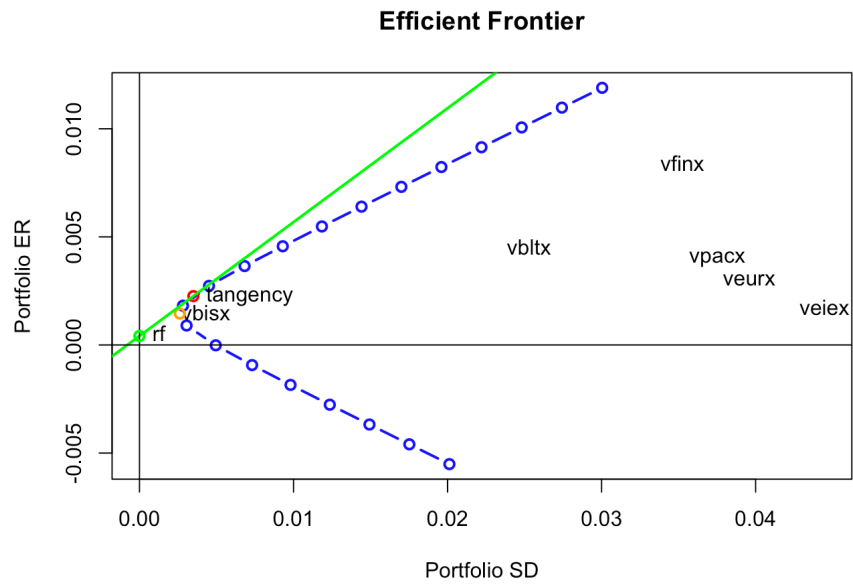
3.7 Compute tangency portfolio

Portfolio Sharpe Ratio		0.527				
Portfolio SD		0.0035				
Portfolio expected return		0.00226				
Portfolio weights	vfinx	veurx	veiex	vbltx	vbisx	vpacx
	0.146	-0.0357	-0.058	-0.0648	1.0039	0.0086



	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Sharpe's Ratio	0.2269113	0.06617832	0.03031994	0.1627416	0.2507149	0.09483252
SE	0.139069	0.1315039	0.132023	0.1402013	0.125763	0.133604

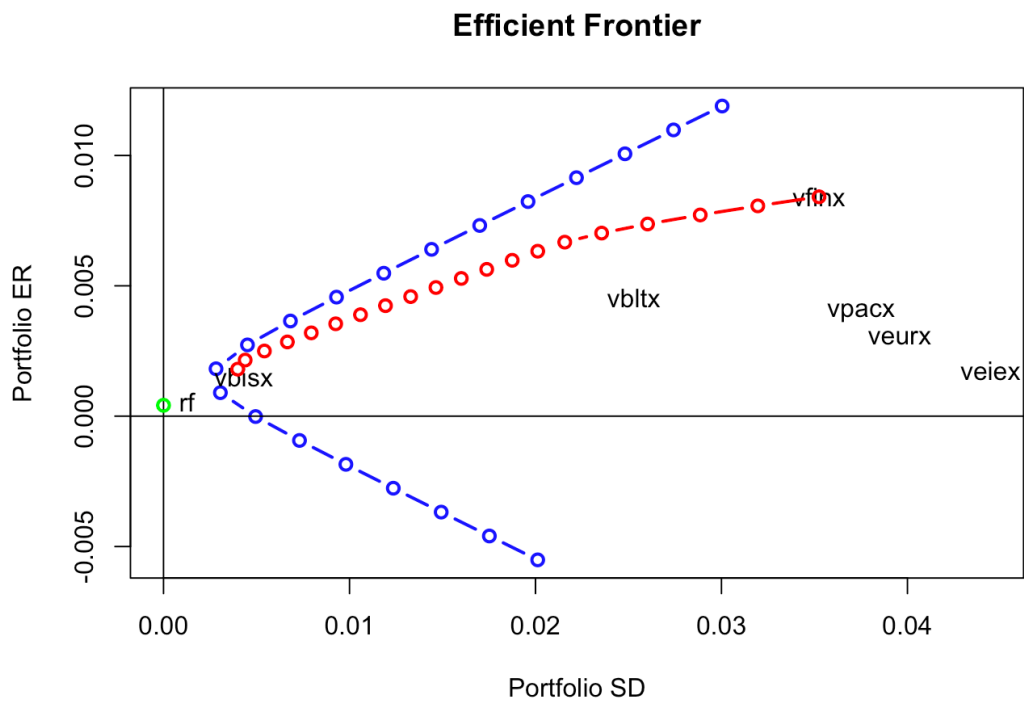
The portfolio has a much higher Sharpe proportion than any individual's Sharpe Ratio. The weights of VERUX, VEIEX, and VBLTX have negative weights.



Annual Portfolio SD	0.012124356
Annual Portfolio expected return	0.02712
Annual Portfolio Sharpe Ratio	1.824426851

The expected return ratio and Sharpe ratio are slightly higher and lower standard deviation than global minimum variance portfolio.

3.8 Efficient portfolio with no short sales



The portfolio has a higher Sharpe proportion than any of the individual resource' Sharpe proportions. This demonstrates for portfolio gives a higher extra hazard premium.

3.9 Compute tangency portfolio with no short sales

Portfolio expected return	0.00221					
Portfolio SD	0.00453					
Portfolio Sharpe Ratio	0.396					
	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Portfolio weights	0.103	0	0	0	0.897	0

The efficient portfolio is 10.3% in VFINX and 89.7% in VBISX.

Annual Portfolio SD	0.01569238
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Annual Portfolio expected return	0.02652
Annual Portfolio Sharpe Ratio	1.371366202

Compared to a tangency portfolio where short-sales are allowed, the short-sales are not allowed has a slightly lower portfolio expected return but higher standard deviation. Also, the short-sales are not allowed has a lower Sharpe ratio than tangency portfolio where short-sales are allowed.

Asset Allocation

4.1 Target 0.5% Return Asset Allocation

Portfolio expected return		0.308				
Portfolio SD		0.0149				
Portfolio Sharpe Ratio		0.005				
	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Portfolio weights	0.371	0	0	0.31	0.32	0

To achieve a target expected return of 6% per year, and VFINX has 37.1% investment, VBLTX has 31% investment, and 32% of VBISX .

4.2 Target monthly SD on this efficient portfolio

Portfolio SD	0.0149
Var 5%	-1951
Var 1%	-2967

4.3 Target 1% Return Asset Allocation & 4.4 Target monthly SD on this efficient

portfolio

```
{r}
target.return <- 0.01
e.port.ns1 <- efficient.portfolio(muhat.vals, cov.mat, target.return,
shorts=FALSE)
e.port.ns1
summary(e.port.ns1,risk.free=rf)
plot(e.port.ns1)

w=100000
q.05.p.ns = e.port.ns1$er + e.port.ns1$sd*qnorm(0.05)
VaR.05.p.ns = q.05.p.ns*w
q.01.p.ns = e.port.ns1$er + e.port.ns1$sd*qnorm(0.01)
VaR.01.p.ns = q.01.p.ns*w
cbind(VaR.05.p.ns,VaR.01.p.ns)
``````
Error in quadprog::solve.QP(Dmat = Dmat, dvec = dvec, Amat =
Amat, bvec = bvec, :
constraints are inconsistent, no solution!
```

There won't be a solution for the target 1% Return Asset Allocation as the expected return is too high that way from the constraints.

## References

标准普尔 500 指数\_百度百科. 百度百科. (n.d.). Retrieved August 19, 2022, from <https://baike.baidu.com/item/%E6%A0%87%E5%87%86%E6%99%AE%E5%B0%94500%E6%8C%87%E6%95%B0/3012967>

“Product Detail -.” Vanguard, [institutional.vanguard.com/investments/product-details/fund/1863](https://institutional.vanguard.com/investments/product-details/fund/1863).